

# On The Super Five Brane Hamiltonian

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## Abstract

The explicit form of the Wess-Zumino term of the PST [1] super 5-brane Lagrangian in 11 dimensions is obtained. A complete canonical analysis for a gauge fixed PST super 5-brane action reveals the expected mixture of first and second class constraints. The canonical Hamiltonian is quadratic in the antisymmetric gauge field and we can recover the bosonic formulation found in [2]. Finally, we find the light cone gauge Hamiltonian for the theory and its stability properties are commented.

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## I. INTRODUCTION

Recently we have study some dynamical aspects for the M5-brane ‘bosonic sector’ [2], it included a complete analysis of the canonical structure of the bosonic sector of the M5-brane starting from the PST action in the gauge where the scalar field is fixed as the world volume time. We found a quadratic dependence on the antisymmetric field for the canonical Hamiltonian. This formulation contains second class constraints that we removed preserving the locality of the field theory in order to construct a master action with first class constraints only. The algebra of the 6 dimensional diffeomorphisms generated by the first class constraints was explicitly given. We constructed the nilpotent BRST charge of the theory and its BRST invariant effective theory. Finally, we obtained its physical Hamiltonian and analyzed its stability properties.

In this work we analyze the canonical structure for the supersymmetric extension of the PST M5-brane action in the same gauge as in the bosonic case. Global  $\epsilon$  supersymmetry and local reparametrization symmetry are manifest in PST Lagrangian and there is a complete proof of the local Kappa symmetry for this action in [3]. In [3] the authors do not need the whole explicit expression of the  $\Omega_6$  term for the Kappa Symmetry proof. Nevertheless, The canonical analysis of the theory requires the explicit form of all terms in the Lagrangian, therefore, the explicit form of the Wess-Zumino term of the PST [1] super five brane Lagrangian is obtained as first step. The canonical study for PST super 5-brane action shows a mixture of first and second class constraints which includes the expected reparametrization generators and second class fermionic constraint which includes kappa symmetry generator, besides the second class constraint associated to the antisymmetric field. The canonical Hamiltonian is quadratic in the antisymmetric gauge field and we can recover the bosonic master formulation found in [2]. In the last section we find the light cone gauge Hamiltonian for the theory and comment on its stability properties.

## II. THE WESS-ZUMINO TERM

The super 5-brane PST Lagrangian is given by  $L = L_1 + L_2 + L_{WZ}$ , with

$$\begin{aligned} L_1 &= 2\sqrt{-\det M_{MN}} \\ L_2 &= \frac{1}{2(\partial a)^2} \tilde{\mathcal{H}}^{MN} \mathcal{H}_{MNP} G^{PL} \partial_L a \\ S_{WZ} &= \int \Omega_6. \end{aligned} \tag{1}$$

where

$$M_{MN} = G_{MN} + i \frac{G_{MP}G_{NL}}{\sqrt{-G(\partial a)^2}} \tilde{\mathcal{H}}^{PL}, \quad (2)$$

and

$$\tilde{\mathcal{H}}^{PL} = \frac{1}{6} \epsilon^{PLMNQR} \mathcal{H}_{MNQ} \partial_R a. \quad (3)$$

Following Schwarz notation . The supersymmetric extension for the antisymmetric field strength  $H = dB$  becomes

$$\mathcal{H} = H - b \quad (4)$$

where

$$b = \frac{1}{6} \bar{\theta} \Gamma_{ab} d\theta [dX^a dX^b + \Pi^a dX^b + \Pi^a \Pi^b] \quad (5)$$

The supercoordinates and the induced supermetric are given by

$$\Pi^a = \Pi_M^a d\sigma^M = (\partial_M X^a + \bar{\theta} \Gamma^a \partial_M \theta) d\sigma^M$$

and

$$G_{MN} = \Pi_M^a \Pi_N^b \eta_{ab}$$

respectively.  $a, b = 0, \dots, 10$  are space time indices and  $M, N = 0, \dots, 5$  are world volume indices. Wess–Zumino term is characterized by the closed seven-form  $I_7 = d\Omega_6$ , where

$$\begin{aligned} I_7 = & -\frac{1}{2} \mathcal{H} \wedge d\bar{\theta} \Gamma_{ab} d\theta \Pi^a \Pi^b \\ & + \frac{1}{60} d\bar{\theta} \Gamma_{abcdef} d\theta \Pi^a \Pi^b \Pi^c \Pi^d \Pi^e \Pi^f. \end{aligned} \quad (6)$$

Note that neither the metric  $G_{\mu\nu}$  nor the scalar field  $a$  appear in  $L_{WZ}$ . Global  $\epsilon$  supersymmetry and local reparametrization symmetry are manifest in PST Lagrangian. Neither PST [1] nor [3] has the explicit expression for the Wess-Zumino term, they only suggest the way to obtain it. In fact, in [3] there is a proof of the local Kappa symmetry for PST action which do not need the whole  $\Omega_6$  expression. However, The canonical study requires the explicit form of all terms in the

Lagrangian, therefore, we deduce it firstly

$$\begin{aligned}
\Omega_6 = & -B \wedge db - \frac{1}{60} \bar{\theta} \Gamma_{abcde} d\theta dX^a dX^b dX^c dX^d dX^e \\
& - \frac{1}{24} d\bar{\theta} \Gamma_{abcde} \theta \wedge d\bar{\theta} \Gamma^e \theta \wedge dX^a dX^b dX^c dX^d \\
& + \frac{1}{12} \bar{\theta} \Gamma_{ab} d\theta \Gamma_{cd} d\bar{\theta} \wedge d\bar{\theta} \Gamma^d \wedge dX^a dX^b dX^c \\
& - \frac{1}{18} d\bar{\theta} \Gamma_{abcde} \theta \wedge d\bar{\theta} \Gamma^e \theta \wedge d\bar{\theta} \Gamma^d \theta \wedge dX^a dX^b dX^c \\
& - \frac{1}{24} \bar{\theta} \Gamma_{cd} d\theta \wedge dX^c dX^d \bar{\theta} \Gamma_{ab} d\theta \wedge \bar{\theta} \Gamma^a d\theta \wedge \bar{\theta} \Gamma^b d\theta \\
& + \frac{1}{24} d\bar{\theta} \Gamma_{abcde} \theta \wedge d\bar{\theta} \Gamma^e \theta \wedge d\bar{\theta} \Gamma^d \theta \wedge d\bar{\theta} \Gamma^c \theta \wedge dX^a dX^b \\
& + \frac{1}{60} d\bar{\theta} \Gamma^b \theta d\bar{\theta} \Gamma^a \theta \wedge d\bar{\theta} \Gamma_{ab} \theta \wedge d\bar{\theta} \Gamma_{cd} \theta \wedge d\bar{\theta} \Gamma^c \theta \wedge dX^d \\
& + \frac{1}{60} d\bar{\theta} \Gamma_{abcde} d\theta \wedge dX^a \wedge \bar{\theta} \Gamma^b d\theta \wedge \bar{\theta} \Gamma^c d\theta \wedge \bar{\theta} \Gamma^d d\theta \wedge \bar{\theta} \Gamma^e d\theta \\
& - \frac{1}{360} d\bar{\theta} \Gamma_{abcde} d\theta \wedge d\bar{\theta} \Gamma^e d\theta \wedge \bar{\theta} \Gamma^d d\theta \wedge \bar{\theta} \Gamma^c d\theta \wedge \bar{\theta} \Gamma^b d\theta \wedge \bar{\theta} \Gamma^a d\theta
\end{aligned}$$

### III. CANONICAL ANALYSIS AND THE LIGHT CONE GAUGE HAMILTONIAN

We consider the gauge in which the scalar field is proportional to the world volume time as in [2] and the spatial world volume indices are now Greek letters. Employ the supersymmetric extension of the ADM parameterization used in [2], the PST  $L_1$  and  $L_2$  become:

$$L_1 = 2n\sqrt{gM} \quad (7)$$

and

$$L_2 = -\frac{1}{4} N^\rho \mathcal{V}_\rho - \frac{1}{2} \tilde{\mathcal{H}}^{\mu\nu} \dot{B}_{\mu\nu} \quad (8)$$

where

$$\begin{aligned}
M &= \det [g_{\mu\nu} + \tilde{\mathcal{H}}_{\mu\nu}], \\
\mathcal{V}_\rho &= \epsilon_{\rho\alpha\beta\sigma\gamma} \tilde{\mathcal{H}}^{\alpha\beta} \tilde{\mathcal{H}}^{\sigma\gamma},
\end{aligned}$$

We have extract from (8) the term independent on  $\dot{B}$ , and added to the Wess-Zumino term in the following way:

$$\Omega_6 - \frac{1}{2} \tilde{\mathcal{H}}^{\mu\nu} b_{[\mu\nu]} \quad (9)$$

It is straightforward to obtain the canonical conjugate momentum of the antisymmetric field, it becomes:

$$P^{\mu\nu} = \tilde{\mathcal{H}}^{\mu\nu}. \quad (10)$$

Equation (10) is a mixture of first and second class constraints.

We can observe that the WZ term does not depend on  $\dot{B}$  and has a linear dependence in  $\dot{\theta}$  and  $\dot{X}^a$ . We can write the canonical conjugate momentum of  $X^a$  and  $\theta$  as:

$$P_a = \tilde{P}_a + f_a(B, X^a, \theta) \quad (11)$$

and

$$S = \tilde{S} + g(B, X^a, \theta) \quad (12)$$

where

$$\begin{aligned} \tilde{P}_a &= \frac{\delta(L_1 + L_2)}{\delta \dot{X}^a} \\ \tilde{S} &= \frac{\delta(L_1 + L_2)}{\delta \dot{\theta}} \end{aligned}$$

The constraints are:

$$\Psi^{\mu\nu} = P^{\mu\nu} - \tilde{\mathcal{H}}^{\mu\nu} = 0 \quad (13)$$

$$\Phi_\alpha = \tilde{P}_a \Pi_\alpha^a + \frac{1}{8} \mathcal{V}_\alpha = 0 \quad (14)$$

$$\Phi = \frac{1}{2} \tilde{P}_a \tilde{P}^a + 2(g + \mathcal{Y}) = 0 \quad (15)$$

$$\bar{\xi} = \tilde{S} + d\bar{\theta} \Gamma^a \tilde{P}_a = 0 \quad (16)$$

where  $\mathcal{Y} = g^{-1} \tilde{\mathcal{H}}^{\mu\nu} \tilde{\mathcal{H}}^{\alpha\beta} g_{\mu\alpha} g_{\nu\beta}$ . Here we have a mixture of first and second class constraints  $\Psi^{\mu\nu}$  is a mixture of first and second class constraints associated to the antisymmetric field.  $\Phi_\alpha$  and  $\Phi$  are the reparametrization generators and  $\xi$  is a mixture of first and second class fermionic constraints which includes the kappa symmetry generator.

The canonical Hamiltonian is the linear combination of the constraints:

$$\mathbf{H} = \Lambda \Phi + \Lambda^\rho \Phi_\rho + \bar{\lambda} \xi + \Xi_{\mu\nu} \Psi^{\mu\nu} \quad (17)$$

Now we are ready to fix the light cone gauge:

$$X^+ = P_0^+ \tau$$

$$P^+ = \sqrt{\omega} P_0^+ \quad (18)$$

and the kappa symmetry:

$$\Gamma^+ \theta = 0 \quad (19)$$

The light cone gauge Hamiltonian then reads:

$$\begin{aligned} \mathbf{H}_{LCG} = & \frac{1}{2\sqrt{\omega}} [\tilde{P}^j \tilde{P}_j + 2(g + \mathcal{Y})] + P_0^+ f^-(B, X^a, \theta) \\ & + \Lambda^{\alpha\beta} \Theta_{\alpha\beta} + \Xi_{\mu\nu} \Psi^{\mu\nu} \end{aligned} \quad (20)$$

where

$$\Theta_{\alpha\beta} = \partial_{[\alpha} \left[ \frac{1}{P_0^+ \sqrt{\omega}} (\tilde{P}_j \Pi_{\beta]}^j + \frac{1}{8} \mathcal{V}_{\beta]} ) - P_0^+ \bar{\theta} \Gamma^- \partial_{\beta]} \theta \right] \quad (21)$$

it contains, together with the fermionic constraint, the generalization of the area preserving constraint.

The second and third terms in (20) represents the physical potential of the super 5-brane. The absolute minimum of this potential is obtained at the configurations satisfying:

$$\begin{aligned} g &= 0, \\ \mathcal{Y} &= 0, \\ P_0^+ &= 0 \text{ or } f^-(B, X^a, \theta) = 0 \end{aligned} \quad (22)$$

If  $\tilde{\mathcal{H}}^{\mu\nu} = 0$ , then the space of physical configurations at which the minimum is obtained, becomes the set of maps  $X^a$  and  $\theta$  from  $\Sigma_5$  to the target space, depending on four linear combinations of the local coordinates. That is, all maps  $X^a$  and  $\theta$  are functions of at most four of them. It is an infinite dimensional space of 1, 2, 3 and 4 branes. The degeneracy of this space is analogous to the one that occurs for the  $D = 11$  supermembrane. There are string like spikes in that case, which are responsible together with supersymmetry for the continuous spectrum of the supermembrane. The degeneracy of the world volume may be pictured as lower p-branes emerging from the world volume which may have free ends or not. It can happen that the other end is plugged into another disconnected sector of the world volume. Such configuration is physically equivalent to the disconnected one, because the tubes do not carry any energy [4].

#### IV. CONCLUSIONS

We have obtained the explicit expression for the Wess-Zumino term which is very important for the canonical study of the super 5-brane theory. We found the canonical and the light cone

gauge Hamiltonian for the super 5-brane theory, starting from the PST action [1] in the gauge where the scalar field is fixed as the world volume time. The canonical constraints include the reparametrization and kappa symmetry generator together with the supersymmetric extension of the second class constraint associated to the antisymmetric field. The results obtained in [2] for the bosonic case of the theory can be recover from the present work. The light cone gauge super potential seems to be unstable. Consequently, the theory should be interpreted as a multiparticle theory as the  $D = 11$  supermbrane.

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